Localization and the interface between quantum mechanics, quantum field theory and quantum gravity II (The search of the interface between QFT and QG)

dedicated to the memory of Rob Clifton

to be published in "Studies in History and Philosophy of Physics"

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November 8, 2010

Contents

Abstract

The main topics of this second part of a two-part essay are some consequences of the phenomenon of *vacuum polarization* as the most important

physical manifestation of *modular localization.* Besides philosophically unexpected consequences, it has led to a new constructive "outside-inwards approach" in which the pointlike fields and the compactly localized operator algebras which they generate only appear from intersecting much simpler algebras localized in noncompact wedge regions whose generators have extremely mild almost free field behavior.

Another consequence of vacuum polarization presented in this essay is the localization entropy near a causal horizon which follows a logarithmically modified area law in which a dimensionless area (the area divided by the square of dR where dR is the thickness of a light sheet) appears. There are arguments that this logarithmically modified area law corresponds to the volume law of the standard heat bath thermal behavior. We also explain the symmetry enhancing effect of holographic projections onto the causal horizon of a region and show that the resulting infinite dimensional symmetry groups contain the Bondi-Metzner-Sachs group. When the first version of this paper was submitted to hep-th it was immediately removed by the moderator and placed on phys. gen without any possibility to cross list, even though its content is foundational QFT. With the intervention of a member of the advisory comitee at least the cross-listing hopefully seems now to be possible.

1 Introduction to the second part

Whereas the first part [1] presented the interface between (relativistic) QM and QFT, this second part focusses on the interface of QFT with gravity, or more precisely on what hitherto was presumed to define this interface[1]. As a result of the very different nature of the localization concept of QFT, a bipartite partition into a subalgebra of a causally closed region and its causal disjoint does not tensor factorize; the sharp localization generates infinitely large vacuum polarization which destroys the quantum mechanical entanglement concept. This is however not the end of the story, since in QFTs with reasonable phase space degrees of freedom one can enforce a tensor factorization via the split construction which re-creates some but not all aspects of quantum mechanical entanglement.

The next section presents this important "splitting" idea. Although there are mathematical examples of QFT which violate the prerequisites for splitting, a physically motivated phase space density of QFT exclude such cases. In particular the existence of the completeness property of asymptotic incoming or outgoing particles in theories with a finite number of particle species imply the splitting property. The third section addresses the most important physical implication of localization, namely *vacuum polarization* and prepares the ground for the presentation of localization entropy in section 4. It is shown that if in an interacting theory one "bangs" with a local operator A onto the vacuum, the so-obtained local vacuum excitation state $A\Omega$ has infinitely many particle/antiparticle components whose analytic continuation determine all formfac-

[1] An example is the concept of entropy associated with horizons, which according to our results in this essay can be formulated and understood in the standard setting of LQP.

tors of A through the crossing relations. Section 5 explains the mathematical/conceptual meaning of holography onto horizons and shows how the loss in information and spacetime symmetry can be reconciled with a huge in conformal symmetry: the holographic projection admits infinite dimensional symmetry groups which contains in particular the classical Bondi-Metzner-Sachs group [22]. The increase of conformal symmetry on the horizon does not help in inverting the holographic projection back towards the reconstruction of the bulk.

The perhaps greatest progress has been the adaptation of the Einstein local covariance principle to the quantum realm [39] which is briefly sketched in section 6. This goes a long way towards the "background independence" which is the would-be "Holy Grail" of the still elusive QG.

The modular theory of which relevants parts for the 2-part paper were presented in part I, plays an important role in connecting the entropy aspects of the heat-bath situation with those of localization. According to our best knowledge the connecting formulae in section 4 are new.

2 The split inclusion

There is one property of LQP which is indispensable for understanding how the quantum mechanical tensor factorization can be reconciled with modular localization: the *split property.*

Definition: *Two monads $\mathcal{A},\mathcal{B}$ are in a split position if the inclusion of monads $\mathcal{A}\subset\mathcal{B}'$ admits an intermediate type I factor $\mathcal{N}$ such that $\mathcal{A}\subset\mathcal{N}\subset\mathcal{B}'$*

Split inclusions are very different from modular inclusions or inclusions with conditional expectations. Their main property is the existence an $\mathcal{N}$-dependent unitarily implemented isomorphism of the $\mathcal{A},\mathcal{B}$ generated operator algebra into the tensor product algebra

$$\mathcal{A}\vee\mathcal{B}\rightarrow\mathcal{A}\otimes\mathcal{B}\subset\mathcal{N}\otimes\mathcal{N}'=B(H) \tag{1}$$

The prerequisite for this factorization in the LQP context is that the monads commute, but it is well-known that local commutativity is not sufficient, the counterexample being two double cones which touch each other at a spacelike boundary [2]. But as soon as one localization region is spacelike separated from the other by a (arbitrary small) spacelike security distance, the interaction-free net satisfies the split property under very general conditions. In [3] the relevant physical property was identified in form of a phase space property (part I, section 4). Unlike QM, the number of degrees of freedom in a finite phase space volume in QFT is not finite, but its infinity is in some sense mild; it is a *nuclear set* for free theories and this nuclearity requirement[2] is then postulated for interacting theories [2]. The physical reason behind this nuclearity requirement is that it allows to show the existence of temperature states once one knows that a QFT exists in the vacuum representation. Even more important its validity prevents the violation of the causal shadow property which states that

[2] A set of vectors is nuclear if it is contained in the range of a trace class operator.

the degrees of freedom in the causal shadow of a spacetime region are the same as those in the original region: $\mathcal{A}(\mathcal{O}") = \mathcal{A}(\mathcal{O})$ which is the algebraic analog of hyperbolic Cauchy propagation. All these properties are formally true in a Lagrangian setting but they constitute a physical safety kit for movements outside the standard quantization parallelism to classical field theory as holography onto horizons, AdS-CFT correspondences etc.

The split property for two securely causally separated algebras has a nice physical interpretation. Let $\mathcal{A} = \mathcal{A}(\mathcal{O}),\ \mathcal{B}' = \mathcal{A}(\check{\mathcal{O}}),\ \mathcal{O} \subset \check{\mathcal{O}}$. Since $\mathcal{N}$ contains $\mathcal{A}$ and is contained in $\mathcal{B}'$ (but without carrying the assignment of a sharp localization between $\mathcal{O}$ and $\check{\mathcal{O}}$), one may imagine $\mathcal{N}$ as an algebra which shares the sharp localization with $\mathcal{A}(\mathcal{O})$ in $\mathcal{O}$, but its localization in the "collar" between $\mathcal{O}$ and $\check{\mathcal{O}}$ is "fuzzy" i.e. the collar subalgebra is like a "haze" which does not really occupy the collar region. This is precisely the region which is conceded to the vacuum polarization cloud in order to spread and thus avoid the infinite compression into the surface of a sharply localized monad. If we take a sequence of $\mathcal{N}'s$ which approach the monad $\mathcal{A}$, the vacuum polarization clouds become infinitely large in such a way that no direct definition of e.g. their energy or entropy content is possible.

The inclusion of the tensor algebra of monads into a type I tensor product (1) looks at first sight like a déjà vu of QM tensor factorization, but there are interesting and important differences. In QM the tensor factorization obtained from the Born localization projector and its complement is automatic since the vacuum of QM (or the ground state of a quantum mechanical zero temperature finite density system) tensor factorizes. In QFT the vacuum does not tensor factorize at all, but there are other states the so-called "split vacuum" states in the Hilbert space which emulate a tensor-factorizing vacuum in the sense that expectation values of operators in $\mathcal{A}(\mathcal{O}) \vee \mathcal{A}(\check{\mathcal{O}}')$ factorize in the split vacuum

$$\langle 0_{split} \left| AB \right| 0_{split} \rangle = \langle 0 \left| A \right| 0 \rangle \langle 0 \left| B \right| 0 \rangle,\ \ A \in \mathcal{A}(\mathcal{O}), B \in \mathcal{A}(\check{\mathcal{O}}') \tag{2}$$

but there remains a huge conceptual difference to the quantum mechanical Born factorization of the "nothing" state. The splitting process requires the supply of energy since the split vacuum has infinite vacuum polarization (with finite mean energy) in the collar region which is spacelike to $\mathcal{O} \vee \check{\mathcal{O}}'$. The physical states of QFT are by definition the states with arbitrary large but finite energy. Their massive particle content is finite but they may contain (as it is the case in QED) infinitely many zero mass particles. In contradistiction to QM these states only tensor-factorize after a spatial split, in which case the reduced vacuum and all finite energy states become thermal Gibbs state with respect to a split-related Hamiltonian. Without the split the $\mathcal{A}(\mathcal{O})$-reduced vacuum state is a singular[3] thermal KMS state.

The problem of physical realizability has not been given much attention in foundational discussions of QM. In QFT this issue is more serious since the situations are much more counter-intuitive, as was shown before with the particle behind the moon argument for the global vacuum. This property is

[3] A singular KMS state denotes a KMS state which is not a Gibbs state.

absent in a split vacuum state; the split defines a barrier, but it is unclear how such split states can be prepared and monitored.

Most foundational properties of QM, as violation of Bell's inequalities, the Schroedinger cat property and many other strong deviations from classical reality can be experimentally verified. This is generally not possible for the vacuum polarization caused properties which result from modular localization simply because macroscopic manifestations are too small. A typical example is the Unruh effect i.e. the thermal manifestation of a uniformly accelerated particle counter in the global vacuum, where the temperature created by an acceleration of 1m/sec is $10^{-19}K$ too small for ever being registered. But for the perception of the reality which underlies LQP the difficulty in registering such effects does not diminish their conceptual importance.

The characterization of the restriction of the global vacuum to a local algebra in terms of a thermal state for a modular Hamiltonian holds, independent of whether the local algebra is a sharply localized monad $\mathcal{A}(\mathcal{O})$ or a type I factor $\mathcal{N}$ contained in a larger sharply localized algebra $\check{\mathcal{O}}$, as in the above splitting construction. The only difference is that the in the second case the KMS state is also a Gibbs state i.e. the Hamiltonian on $\mathcal{N}$ has a discrete spectrum (in case $\check{\mathcal{O}}$ is compact). This thermal reinterpretation of reduced states does not only hold for the vacuum, but applies to all states which are of physical relevance in particle physics i.e. to all finite energy states for which the Reeh-Schlieder theorem applies.

Since KMS states on type I factors are Gibbs states, there exists a density matrix. Therefore these Gibbs state can have a finite energy and entropy content which for monads is impossible. But a monad may be approximated by a sequence of type I factors in complete analogy to the thermodynamic limit. In fact the thermodynamic limit is the only place where a monad algebra appears in a QM setting; an indication that this limit is accompanied by a qualitative change is the fact that one looses the density matrix nature of the Gibbs state which changes to a more singular KMS state which simply does not exist on quantum mechanical type I algebras. A related fact is the breakdown of the tensor factorization into physical degrees of freedom and their "shadow world" in the thermodynamic limit. This implies that the "Thermofield formalism" is only applicable for the boxed QFT (type I), it looses its meaning in the thermodynamic limit when the state becomes a singular KMS state and the algebra turns into a monad.

The structural difference can be traced back to the nature of modular Hamiltonians. Whereas for a monad the modular Hamiltonian has continuous spectrum[4] and hence an ill-defined (infinite) value of energy and entropy, this is not the case for the $\mathcal{N}$-associated density matrix constructed from the split situation. So the way out is obvious, one must imitate the thermodynamic limit by constructing a sequence of type I factors (a "funnel" $N_i \supset \mathcal{A}(\mathcal{O})$ by tightening the split) which converge from the outside towards the monad; equivalently one

[4]A typical example is the thermodynamic V$\rightarrow \infty$ limit in which the discrete spectrum of the Hamiltonian in a box turns into a continuous spectrum and the Hamiltonian becomes the modular Hamiltonian on a monad. Gibbs state.

may approximate from the inside.

In the next section the split limit, in which a double-cone localized monad is approximated by a sequence of type I_∞ factors $\mathcal{N}_i$, will be presented. In this case the modular group of $(\mathcal{N}_i, \Omega)$ leads to a Gibbs situation i.e. the restriction of the vacuum to the algebra $\mathcal{N}_i$ is a Gibbs state at the same modular temperature as the that associated with the restriction to the monad. The main distinction to the standard heat bath situation is that Hamiltonians which result from restricting the vacuum to an $\mathcal{N}_i$ obtained from the split construction is constructed according to very different principles from that of Gibbs states in a finite quantization box with the help of the Hamiltonian which describes the time development in a quantum mechanical inertial frame. Nevertheless for two-dimensional chiral theories there exists a rigorous relation between the two kinds of thermal behavior: *the inverse Unruh effect.* The localization-caused thermal manifestation in a chiral theory is related by a conformal trnaformation to the thermodynamic limit of a one-dimensional global heat bath inertial system [21][19]. There are presently no reliable computational techniques for dealing with modular Hamiltonians and their split approximands, although there is no lack of mathematical precision in *defining* these objects. In the next section we will try to overcome this situation by invoking geometrical arguments leading to a localization of the vacuum polarization cloud inside a *light-sheet.*

The split property does not hold in all axiomatic models of LQP but there are rather good arguments that it is valid in "physical" models which share a physically relevant property with Lagrangian models. What is important is not the Lagrangian quantization but rather the causal timelike propagation aspect which historically has been referred to as the time-slice property [4] or the causal shadow property. This property is violated in theories with too many phase space degrees of freedom.

The the study of what constitutes a physical phase space density started in the 60s [5] when for the first time it was shown that the implementation of the relativistic causality principle requires a bigger cardinality of phasespace degree of freedoms than in QM. Whereas the number of degrees of freedom per unit cell in phase space is finite, these authors found the cardinality in QFT is infinite in the sense of being a compact set. This result was later sharpened to *nuclear* $<$ *compact* [6] and various important properties were shown to be consequences, among them the existence of thermal states for all temperatures and the splitting property [20]. The implies the absence of an unwanted Hagedorn temperature as it occurs in infinite component QFT with too many degrees of freedom as string theories. It is easy to invent QFTs which violate this bounds in the sense of having too many degrees of freedom, the generalized free fields with suitably increasing Kallén-Lehmann density are the simplest examples.

Unfortunately the knowledge about these concepts got lost, otherwise how can one explain that a worldwide community works on a narrow subject as the AdS_5-CFT_4 correspondence with more than 6000 contributions without becoming aware that it is structurally impossible to have theories with physical degrees of freedom on both sides of this correspondence? The naive expectation

(backed up by unconvincing calculations outside any mathematical control) in these papers are in contradiction with established facts[5]. One can safely assume that this knowledge has been lost and even worse, that those few correct papers which have been published on this correspondence [26][27][8] are beyond conceptual reach of the typical particle physicist with a contemporary (post string theory) quantum field theoretical background. If one looks at the thousands of publication on this matter one cannot escape the conclusion that the deep conceptual knowledge of the past has been replaced by a more metaphoric mode of thinking in the pursuit of an ultra-reductionist theory of everything (TOE).

In the above form the positioning of monads aims at characterizing LQP in Minkowski spacetime. This begs the question whether there is a generalization to curved spacetime (CST). A very special exploratory attempt in this direction would be to investigate the Diff(S^1) symmetries beyond the Moebius group in chiral theories for their possible modular origin in terms of positioning monads relative to reference states which are different from the vacuum. Since the *extended* chiral theories which result from null-surface holography (and not from chiral projections of a two-dimensional conformal QFT) seem to have great constructive potential, this question could be of practical interest.

I expect that by pursuing the algebraization of QFT in CST according to the positioning of monads viewpoint, one will learn important lessons about the still unknown QFT/QG interface. It would be of great interest to understand whether the isometric isomorphisms related to the local covariance principle (see last section) have modular roots similar to symmetries of the vacuum in Minkowski QFT. A conservative approach which explores unknown aspects of QFT while staying firmly rooted in known principles seems to be the most promising path for pushing the borderline of QFT in CST further towards the still unknown QFT-QG interface.

3 Localization-induced vacuum polarization, present view and history

The phenomenon of vacuum polarization has been the point of departure of many metaphors of which the vacuum in QFT as a "steaming broil" is perhaps the best known because it occasionally even entered textbooks. In order to support this image the appeal to a short time violation of the energy conservation allowed by the uncertainty relation was made. A less metaphoric view comes from locally "banging" on the vacuum i.e. applying a compactly localized operator to the vacuum state. Such a state is characterized by its n-particle matrix elements for all n and these n-particle vacuum polarization components are in turn special boundary values of an analytic n-particle master function whose

[5] The sharing of the SO(4,2) conformal symmetry prevents a dilution of degrees of freedom in passing from the higher- to the lower- dimensional theory. Starting from a standard conformal QFT (the supersymmetric N=4 Yang-Mills theory is a potential candidate) the degrees of freedom transplanted to the higher dimensional AdS spacetime are not sufficient to "fill" the AdS bulk and remain hovering at the boundary.

different out-in particle distributions obtained from the vacuum polarization component by crossing are the formfactors of A.

$$A\Omega \simeq \{\langle 0|\, A\, |p_1, ...p_n\rangle^{in}\}_n \tag{3}$$
$$\overset{cros\sin g}{\rightarrow} \{^{out}\langle -\bar{p}_{k+1}, ... - \bar{p}_n|\, A\, |p_1, ...p_k\rangle^{in}\}_n$$

where the negative mass shell momenta $-\bar{p}$ denotes the analytic continuation which is part of the crossing process and the bar is the reminder that the particle is an antiparticle to the original particle (this can be omitted in case of self-conjugate particles). In case of $A \in \mathcal{A}(\mathcal{O})$ being the identity operator there is no "banging" onto the vacuum; in that case we are dealing with the S-matrix for which many matrix-elements vanish as a result of the energy momentum conservation between the particles without the feeding in from the localized operator A. In particular the S-matrix is free of vacuum polarization clouds, since the identity leaves the vacuum invariant and does not bang.

The crossing property is one of the deepest characteristics linking fields and particles. It had been known for a long time that there are no compactly localized operators which applied to the vacuum generate a one-particle state without an admixture of a vacuum polarization cloud except if the theory is that of a free field[6]. One-particle operators exist as in- or out- operators (or unitary transforms thereof) in the full algebra $B(H)$, but they have a very nonlocal relation with respect to the localized operators. The wedge region is a borderline localization in that there exist wedge-localized operators which *even in the presence of interactions create polarization-free one-particle* (and also multiparticle incoming) *states*. These operators are not identical to the incoming creation/annihilation operators but they share the same Reeh-Schlieder domain for wedge localization. The wedge restricted vacuum state is a KMS state and the KMS property is intimately related to the one-step crossing [44]. A proof of crossing from these localization properties would go beyond the more modest aims of this paper.

Whereas in QM, relativistic or not, one has great liberty in manipulating interaction potentials without leaving the setting of QM so that almost any prescibed outcome can be accommodated, this is not the case in QFT. There the locality principle is very restrictive and this tightness even show up in theorems about the S-matrix as the Aks theorem saying that in a 4-dimensional QFT nontrivial elastic scattering is not possible without the presence of inelastic components [11]. For the formfactors the statement (3) has a more popular stronger form which, although probably provable, has according to my best knowledge presently the status of a fact being supported by experience. This is the apparent validity of a kind of benevolent Murphy's *law: all couplings of local operators to other channels (in the case of formfactors multiparticle channels) which are not forbidden by superselection rules actually do occur.* Of course one needs to "bang onto the vacuum", there is no "boiling soup" in a an inertial frame without heating the "vacuum stove".

[6] Within the Wightman setting this has been known as the Jost-Schroer theorem [9]. A stronger form was recently proven in the algebraic setting [10].

The formfactor aspect of a local operator is perhaps the best QFT illustration of Murphy's law to particle physics. This tight coupling of channels through the realization of the locality principle is both a blessing and a curse. It attributes a holistic structure to QFT which on the one hand aggravates the strategy to divide the difficult problem of (nonperturbative) model construction into easier pieces, but on the other hand is the main reason why this theory is much more fundamental than QM. In particular it does not support the presently fashionable idea of "effective" QFT in which the holistic aspect is largely ignored and which is eulogized when it does give a wanted result and disregarded if it does not. Some interesting and pertinent remarks about the importance of the holistic point of view in connection with the problem of the energy density of the cosmological reference state can be found in [12]

Vacuum polarization as a concomitant phenomenon of QFT was discovered a long time before the role of locality took the center stage. It is interesting to reformulate Heisenberg's first observation in a more modern context by defining partial charges by limiting the charged region with the help of smooth test function. In Heisenberg's more formal setting the partial charge of a free conserved current in a spatial volume V is defined as

$$\begin{aligned} Q_V &= \int_V j_0(x,t) d^3x \\ j_\mu(x,t) &=: \phi^*(x,t) \overset{\leftrightarrow}{\partial}_\mu \phi(x,t) : \end{aligned} \tag{4}$$

Introducing a momentum space cutoff, the norm of $Q_V \left|0\right\rangle$ turns out to diverge quadratically which together with the dimensionlessness of Q is tied to the area proportionality. Hence already on the basis of a crude dimensional reasoning one finds an area proportionality of vacuum polarization. The cutoff was the prize to pay for ignoring the singular nature of the current which is really not an operator but rather an operator-valued distribution.

The modern remedy is to take care of the divergence by treating the singular current as an operator-valued distribution. Such calculations have been done in the 60s [13][14] by using spacetime test functions which regularize the delta function at coalescing times and are equal to one inside the ball with radius R and fall off to zero smoothly between R and $R+\Delta R$. Using the conservation law of the current one can then show [13] that the action of the regularized partial charge on the vacuum is compressed to the shell $(R, R+\Delta R)$ and diverges quadratically with $\Delta R \rightarrow 0$ i.e. As expected, the vacuum fluctuations vanish weakly as $R \rightarrow \infty$ (even strongly by enlarging the time smearing support of g together with R [15]) i.e. the limit converges independent of the special test function weakly[7] to the global charge operator

$$\lim_{R\rightarrow\infty} \int f_{R,\Delta R}(\vec{x}) g(t) j_0(x,t) d^4x = Q \tag{5}$$

[7]Although the norm diverges, the inner product of $Q_R \left|0\right\rangle$ with localized states converges to zero in compliance with the zero charge of the vacuum.

With other words in the limit of global charges the vacuum polarization drops out together with the test function dependence.

The interesting question in the context of the present section is the question of what is the $R, \Delta R$ dependence when $\Delta R \to 0$. The answer depends on the dimension of spacetime and the leading divergence can be calculated for free currents in massless theories. The simplest case is that of a chiral current which is localized on a light ray

$$\langle j(x)j(x')\rangle \simeq \frac{1}{(x-x'+i\varepsilon)^2} \tag{6}$$

$$\|Q(g_{R,\Delta R})\Omega\| \,\tilde{}\, \ln\frac{R}{\Delta R}$$

i.e. different from QM the dimensionless *partial* charges diverges in QFT, the first manifestation of vacuum polarization as first observed by Heisenberg. In higher dimensional QFT the logarithmic behavior is modified by powers in $\frac{R}{\Delta R}$; in particular in d=1+3 one obtains an (logarithmically corrected) area law

$$\|Q_{R,\Delta R}\Omega\| \,\tilde{}\, (\frac{R}{\Delta R})^2 \ln\frac{R}{\Delta R} \tag{7}$$

The vacuum polarization-caused $\Delta R \to 0$ divergencies of this partial charge operator are preempted in a certain sense by the behavior of the dimensionless localization-entropy; however despite similarities the computation of the latter is conceptually more involved. The reason is that the entropy is inherently nonlocal in the sense that it cannot be obtained by a integrating a pointlike conserved current (or any other operator) but rather encodes a holistic aspect of an entire algebra. Nevertheless the *splitting property* (for a description of its history see [2]) is in a certain sense the algebraic analog of the test-function smearing on individual field operators.

Entropy in QM is an information theoretical concept which measures the degree of entanglement. The standard situation is bipartite spatial subdivision of a global system so that global pure states become entangled with respect to the subdivision i.e. they can be written as a superposition of tensor product states. The entropy is than a number computed according to the von Neumann definition from the reduced impure state which results in the standard way from averaging over the opposite component in one of the tensor factors.

The traditional quantum mechanical way to compute entanglement entropy was applied to QFT of a halfspace (a Rindler wedge in spacetime) for a system of free fields in a influential 1984 paper [16]. The authors started from the assumption that the total Hilbert space factorizes in that belonging to the halfspace QFT and its opposite. The calculation is ultraviolet divergent and after introducing a momentum space cutoff κ, the authors showed that the cutoff dependence is consistent with an area behavior.

$$S/A = C\kappa^2 \tag{8}$$

where in the conformal case C is a constant, κ is a momentum space cutoff and S/A denotes the surface density of entropy. The method of computation is again

the integration over the degrees of freedom of the complement region and the extraction of the entropy from the resulting reduced density matrix state whose degree of impurity encodes the measure of the inside/outside entanglement.

This calculation should be seen in analogy with Heisenberg's momentum space cut-off calculation of vacuum polarization in the partial charge (4). In both cases the starting formula is morally correct but factually wrong. Neither is the partial charge inside a region defined by a volume integral nor do (as we know from discussions in previous sections) global states in QFT permit an inside/outside factorization. These incorrect assumptions create the divergencies which are then kept under the lid by the popular emergency kit of QFT: momentum space cutoff. In both cases dimensional arguments lead to an area proportionality (with logarithmic corrections possibly escaping the consideration).

The main advantage of the present spacetime approach versus a momentum space cutoff argument is that the split property teaches us that the vacuum polarization cloud hovers near the horizon in the split region characterized by the sheet size ΔR. The divergence for $\Delta R \rightarrow 0$ indicates in no way a conceptual inconsistency or shortcoming of QFT which must must be overcome with the help of quantum gravity. With other words the localization entropy is a notion within a specified QFT, it does not need any reference with respect to an ill-defined nonlocal "cutoff theory"[8]. There are simply some quantities whose sharp localization causes divergencies but whose global value is perfectly finite; for the global charge it is the finite value carried by one or several particles and in the case of the entropy its global value in the ground state vanishes.

4 The elusive concept of localization entropy

Let us first apply the previously presented split idea to a two-dimensional conformal QFT in which case the double cone is a two-dimensional spacetime region consisting of the forward and backward causal shadow of a line of length L at $t = 0$ sitting inside larger cone obtained by augmenting the baseline on both sides by ΔL. As a result of the assumed conformal invariance of the theory, the canonical split algebra inherits this invariance and hence the entropy Ent of the canonical split algebra can only be a function of the cross ratio of the 4 points characterizing the split inclusion

$$
\begin{aligned}
Ent &= -tr\rho ln\rho = f\left(\frac{(d-a)\,(c-b)}{(b-a)\,(d-c)}\right) \qquad (9) \\
with\ a &< b < c < d = -L - \Delta L < -L < L < L + \Delta L
\end{aligned}
$$

where for conceptual clarity we wrote the formula for generic position of 4 points. Our main interest is to determine the leading behavior of f in the limit $\Delta L \rightarrow 0$ (two pairs of points coalesce) which is the analog of the thermodynamic limit $V \rightarrow \infty$ for heat bath thermal systems.

[8] The concept of a theory with a cutoff cannot even be defined in the presence of interactions, even if one limits the construction to the family of soluble factorizing models.

The asymptotic estimate for $\Delta L \to 0$ can be carried out with an algebraic version of the *replica trick* which uses the cyclic orbifold construction in [17]. First we write the entropy in the form

$$Ent = -\frac{d}{dn} tr \rho^n |_{n=1}, \ \rho \in M_{can} \subset \mathcal{A}(L + \Delta L) \tag{10}$$

Then one uses again the split property, this time to map the n-fold tensor product of $\mathcal{A}(L+\Delta L)$ from the replica trick into the algebra of the compactified line $\dot{R} = S^1$ with the help of the n^{th} root function $\sqrt[n]{z}$. The part which is invariant under the cyclic permutation of the n tensor factors defines the algebraic version [17] of the replica trick. The transformation properties under Moebius group are now given in terms of the following subgroup of DiffS^1 written formally as

$$\sqrt[n]{\frac{\alpha z^n + \beta}{\bar{\beta} z^n + \bar{\alpha}}}, \ L'_{\pm n} = \frac{1}{n} L_{\pm n}, \ L'_0 = L_0 + \frac{n^2-1}{24n} c \tag{11}$$
$$\dim_{\min} = \frac{n^2-1}{24n} c$$

where the first line is the natural embedding of the n-fold covering of Moeb in $\mathrm{diff}(\mathrm{S}^1)$ and the corresponding formula for the generators in terms of the Virasoro generators. As a consequence the minimal L'_0 value (spin, anomalous dimension) is the one in the second line. With this additional information coming from representation theory we are able to determine at least the singular behavior of f for coalescing points $b \to a$, $d \to c$

$$Ent_{sing} = -lim_{n \to 1} \frac{d}{dn} \left[\frac{(d-a)(c-b)}{(b-a)(d-c)} \right]^{\frac{n^2-1}{24n}} = \frac{c}{12} ln \frac{(d-a)(c-b)}{(b-a)(d-c)} \tag{12}$$

Since the function is only defined at integer n, one needs to invoke Carlson's theorem. Apart from the split setting the calculation follows the same steps as entropy calculations in condensed matter physics [18] which is based on certain assumed properties of functional integrals which permit the avoidance of momentum space cutoffs.

The resulting leading contribution to the entropy reads

$$Ent_{sing} = \frac{c}{12} \ln \frac{(d-a)(c-b)}{(b-a)(d-c)} = \frac{c}{12} ln \frac{L(L+\Delta L)}{(\Lambda L)^2} \tag{13}$$

where c in typical cases is the Virasoro constant (which appears also in the chiral holographic lightray projection).

This result was previously [19] obtained by the "inverse Unruh effect" for chiral theories which is a theorem stating that for a conformal QFT on a lightlike line the KMS state obtained by restricting the vacuum to the algebra of an interval is unitarily equivalent to a global heat bath temperature state for a certain (geometry-dependent) value of the temperature. The chiral inverse Unruh effect involves a change of length parametrization; the length proportionality

of the heat bath entropy (the well known volume factor) is transformed into a logarithmic length measure. The inverse Unruh effect has only been established in chiral QFT, but it points towards a question of significant conceptual and philosophical importance: is there a structural relation between heat bath and localization-caused thermal behavior or are do they represent two unrelated physical phenomena ?

On expects the two monads to behave in the same way after reparametrizing in a way which accounts for the different spacetime aspects of the two monads and their different approximations by type I factors. In the thermodynamic case the monad is approached by type I algebras of Gibbs states on systems in a box of volume V in the limit $V \rightarrow \infty$ whereas approximations of the $\mathcal{A}(\mathcal{O})$ monade is done by the type I factors obtained from the split property. since the vacuum restricted to split type I factors also turn out to be thermal (with respect to the modular Hamiltonian) one expects a universality in the two kinds of thermal behavior. Therefore the relevant question is: can the volume divergencies of the heat bath thermodynamic entropy be set in relation to the $\Delta R \rightarrow 0$ divergence in the area behavior (possible modified by a logarithm) caused by vacuum polarization as in (7) ? And do all dimensionless localized objects have the same leading divergence for $\Delta R \rightarrow 0$. Since QFT does not know any frame-independent position operator ("effective" substitutes have no place in conceptual arguments) the question arises whether QFT can offer an analog to the Heisenberg uncertainty relation. A universal relation between the leading entropy/energy increase with the sharpness of localization is as close as one could come.

This thermal universality hypothesis would suggest the following correspondence between the heat bath and the localization entropy

$$((kT)^{n-1}V_{n-1})_{T=2\pi} \simeq \frac{R^{n-2}}{(\Delta R)^{n-2}} ln \frac{R^2}{(\Delta R)^2} \tag{14}$$

$$Ent(h.b.)_{T=2\pi} = Ent(loc) \tag{15}$$

where the first line expresses the reparametrization of the dimensionless (n-1)-volume factor in terms of a dimensionless logarithmically corrected dimensionless area factor. Since localization thermality is a phenomenon of modular theory which does not know anything about kT, the dimensionless area is obtained from the thickness of the light slice which appeared already as a ΔL in the logarithmic divergence for n=2, the case in which we presented an rigorous proof based on the chiral inverse Unruh effect. The above equality for the entropies means in particlar that the two matter dependent finite constants in front of the leading divergencies are the same if we use the same quantum matter for the heat bath and the localization situation. For n>2 the relation is conjectural but its violation would cause serious problems in our understanding of QFT. Naturally this correspondence can only be expected for the leading term in the thermodynamic limit $V \rightarrow \infty$ respectively in the "funnel" limit $\Delta R \rightarrow 0$ of decreasing split distance.

A mathematical proof would amount to the calculation of the von Neumann entropy of the density matrix ρ which results from the restriction of the vacuum to the split tensor factor, a task which goes beyond the present computational abilities in QFT. However it is possible to present some more details supporting details about the geometrical aspects of the situation which are closely related to the leading $\Delta R \to 0$ behavior of the dimensionless partial charge operators (7) caused by the vacuum polarization cloud.

Compared with the chiral models in the beginning of this section which can be controlled quite elegantly with the replica method, the question of higher dimensional localization entropy looks more involved. In terms of inclusions and relative commutants the funnel approximation to the double cone situation is described in terms of the following split inclusion [20]

$$\begin{aligned} &\mathcal{A}(\mathcal{D}(R)) \subset \mathcal{N} \subset \mathcal{A}(\mathcal{D}(R+\Delta R)) \\ &\mathcal{A}(ring) \equiv \mathcal{A}(\mathcal{D}(R))' \cap \mathcal{A}(\mathcal{D}(R+\Delta R)), \\ &\mathcal{N} = \mathcal{A}(\mathcal{D}(R)) \vee J_{ring}\mathcal{A}(\mathcal{D}(R))J_{ring} \end{aligned} \tag{16}$$

where $\mathcal{N}$ is the canonically associated type I algebra in terms of which there is tensor factorization as in (1) and canonical means that there is an explicit formula in terms of the double cone algebra localized symmetrically around the orgin with radius R and a larger one with radius $R+\Delta R$. The canonical formula for $\mathcal{N}$ is written in the third line where J_{ring} is the modular reflection for the ring algebra defined in the second line. Note that this ring region is contained in a *light sheet* between the two horizons of $\mathcal{D}(R)$ and $\mathcal{D}(R+\Delta R)$.

The crucial geometric input which leads to the desired result is the realization that the relevant part for the area-like behavior is the fact that the vacuum on $\mathcal{N}$ only contributes in the ring region since on $\mathcal{D}(R)$ it is indistinguisable from the old vacuum. The ring region is proportional to the area, and allowing for the previously established logarithmic behavior in lightlike direction, one ends up with [22]

$$Ent(\mathcal{D}(R)) \overset{\Delta R \to 0}{\simeq} C(n)\frac{R^{n-2}}{(\Delta R)^{n-2}}\frac{c}{12}ln\frac{R(R+\Delta R)}{(\Delta R)^2},\ C(0)=1 \tag{17}$$

where the logarithm is the only singularity in chiral conformal (n=2) models. The naive geometrical argument would favor the dimensionless area law involving the ring size ΔR without the logarithm, whereas the *presence of the logarithm can be viewed as representing a lightlike length factor* which according to the chiral inverse Unruh effect is mapped into a logarithmic divergence. In this way of counting there is a perfect match with the (n-1)-volume factor apart form the fact that one length factor has to be mapped into a logarithm.

The result contradicts the popular folklore that QFT is incomparible with an area behavior, which is sometimes used delineate QFT in CST from QG. The presence of the logarithm is important for our conjecture of thermal universality [22] which would find its most perfect expression in the existence of a yet hypothetical higher dimensional inverse Unruh effect (more in the concluding remarks); this remains an interesting problem for future research.

5 Holography onto horizons, BMS symmetry enhancement

The special role of null-surfaces as causal boundaries, which define places around which vacuum polarization clouds form, suggests that there may be more to expect if one only could make *QFT on a light-front* a conceptually and mathematically valid concept. That this can be indeed achieved is the result of holography. Holography clarifies most of the problem which were raised by its predecessor, the "lightcone quantization" and explains why the older method failed. One of the reasons has to do with short distance behavior since the naive restriction of fields to space- or light-like submanifolds require the validity of the canonical quantization formalism i.e. a short distance dimension not worse than sdd=1, even though the there is no such restriction on the dimension of chiral fields living on a lightray.

However the causal localization principle in its algebraic formulation permits to attach to each region the algebra of its causal shadow. For null-surfaces the situation with respect to pointlike generators improves. In that case the observable algebras indexed by regions on the lightfront are really pointlike field-generated and the field generators are transversely extended chiral observable fields $C(x,\mathbf{x})$ where x denotes the lightlike coordinate on the lightfront and $\mathbf{x}$ parametrizes the n-2 dimensional transverse submanifold. The absence of transverse vacuum polarization would suggest to expect their commutation relations to be of the form

$$[C_i(x_1,\mathbf{x}_1),C_j(x_2,\mathbf{x}_2)]=\delta(\mathbf{x}_1-\mathbf{x}_2)\sum_{k=0}^{m}\delta^{(k)}(x_1-x_2)\check{C}_k(x_1,\mathbf{x}_1) \tag{18}$$

where the number m of operator contributions on the right depends on the scale dimensions of the two operators on the left hand side. The transverse delta function expresses the absence of transverse vacuum polarization which is a rigorous-model independent result of the algebraic setting [21]. As for standard chiral fields the scale dimensions are unlimited (no restriction to canonicity as for equal time commutations)[9]. The $\check{C}(x,\mathbf{x})$ denote $\mathbf{x}$ dependent chiral fields of which has to know (as a consequence of the absence of transverse vacuum fluctuations) only the product structure in $x,\ x'$ at the same $\mathbf{x}$ which Such a commutation relation, with the exception of d=1+1 where there is no transverse dependence, can however not be quite correct for composite fields as a simple free field calculation for $:A^2(x):$ shows [23] in which case an ill-defined square of a delta function appears (see below). This is perhaps a reminder that one should not aim for the holographic projection of individual pointlike fields in any literal sense, but rather seek pointlike generators of the holographically projected algebra according to: bulk fields$\rightarrow$bulk local algebras$\rightarrow$ holographic projection$\rightarrow$ construction of pointlike generators.

[9]There can be higher derivatives in the transverse direction but they are always even whereas the light-like delta functions are odd.

The modular localization theory plays a crucial role in the construction of a local net on the lightfront and its generating fields and for this reason one must start with operator algebras which is in a standard position with respect to the vacuum. Since the full lightfront algebra is identical to the global algebra on Minkowski spacetime, one must start with a subregion on the lightfront and the largest such region is half the lightfront whose causal completion is the wedge (so that it can be seen as the wedge´s causal (upper) horizon $H(W)$[10])

$$\mathcal{A}(W) = \mathcal{A}(H(W)) \subset B(H) \tag{19}$$

It is very important to avoid to project more into this equation than what is actually written: this equality refers only to the position of the two algebras within the full algebra $B(H)$; it does not refer to their local substructure. The latter would be very different indeed; the local substructure consisting of the net of (arbitrarily small) double cones inside $\mathcal{A}(W)$ and that on $H(W)$ have no direct relation.

The local substructure on the horizon $\mathcal{A}(H(W))$ is obtained by intersecting different W algebras which have their horizons on the same lightfront. In 4-dimensional Minkowski spacetime they are connected by a 7-parametric subgroup of the 10-parametric Poincaré group containing: 5 transformations which leave W invariant (the boost, 1 lightlike translation, 2 transverse translations, 1 transverse rotation) and 2 transformations which change the edge of W (the two "translations" in Wigner's *Little Group*). This 7-parametric subgroup is precisely the invariance group of the lightfront, but as a consequence of the absence of transverse vacuum polarization of QFTs on null surfaces, the loss of symmetry is more than compensated for by a gigantic symmetry gain leading to an infinite parametric symmetry containing the Bondi-Metzner-Sachs group.

Although the net structure of the bulk determines that on a lightfront, the inverse is not true, it is not possible to construct the net structure of $\mathcal{A}(W)$ from that of $\mathcal{A}(H(W))$. The additional information beyond the intrinsic data of the $\mathcal{A}(H(W))$ net which will secure unique inversion can have different appearance: Poincaré transformations or characteristic propagation laws off $H(W)$ or the relative positioning (forming a kind of *algebraic GPS system*) of not more than three lightfronts in different appropriate relative positions. The loss of information, of phase space degrees of freedom and of symmetries (those which transform out of the null surface) are all interconnected and related to the projective nature of holography onto horizons a projection. The only known case of a bona fide correspondence is the AdS_n-CFT_{n-1} isomorphism in which case the symmetry groups are identical.

For free fields the construction can be done explicitly. Since it is quite interesting and sheds some light on why the holography works where the old lightcone quantization did not succeed, the remainder of this section will be used to present the free field holography[11].

[10] This is the quantum version of causal propagation with characteristic data on $H(W)$. A smaller region on LF does not cast a causal shadow.

[11] I am indebted to Henning Rehren who informed me that similar idea can be traced back

The crucial property, which permits a direct holographic projection, is the mass shell representation of a free scalar field

$$A(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int (e^{ipx} a^*(p) \frac{d^3p}{2p_0} + h.c.) \tag{20}$$

With the help of this representation one can directly pass to the lightfront by using lightfront adapted coordinates $x_\pm = x^0 \pm x^3$, $\mathbf{x}$, in which the lightfront limit $x_- = 0$ can be taken without causing a divergence in the p-integration. Using a p-parametrization in terms of the wedge-related hyperbolic angle θ : $p_\pm = p^0 + p^3 \simeq e^{\mp\theta}$, $\mathbf{p}$ the $x_- = 0$ restriction of $A(x)$

$$\begin{aligned} A_{LF}(x_+, \mathbf{x}) &\simeq \int \left(e^{i(p_-(\theta)x_+ + i\mathbf{px}} a^*(\theta, \mathbf{p}) d\theta d\mathbf{p} + h.c. \right) \\ \left\langle \partial_{x_+} A_{LF}(x_+, \mathbf{x}) \partial_{x\prime_+} A_{LF}(x'_+, \mathbf{x}') \right\rangle &\simeq \frac{1}{\left(x_+ - x'_+ + i\varepsilon\right)^2} \cdot \delta(\mathbf{x} - \mathbf{x}') \\ \left[\partial_{x_+} A_{LF}(x_+, \mathbf{x}), \partial_{x\prime_+} A_{LF}(x'_+, \mathbf{x}') \right] &\simeq \delta'(x_+ - x'_+) \delta(\mathbf{x} - \mathbf{x}') \end{aligned} \tag{21}$$

The justification for this formal manipulation uses the fact that the equivalence class of test function which have the same restriction $\tilde{f}|_{H_m}$ to the mass hyperboloid of mass m is mapped to a unique test function f_{LF} on the lightfront [25][21]. One easily verifies the identity $A(f) = A(\{f\}) = A_{LF}(f_{LF})$. But note also that this identity does not mean that the A_{LF} generator can be used to construct the localization structure from that of a characteristic initial value problem which, concerning localization issues, is very different from a Cauchy initial value problem. Even classically the lightfront-bulk relation is primarily one between symplectic subspaces of the global symplectic space of all classical waves, rather than relations between individual solutions.

The restriction to transverse or lightlike compact data does not improve the localization within the wedge, it only causes "fuzziness", i.e. lack of reconvertibility of an algebraic automorphism to a geometric diffeomorphism. So algebraic holography from a wedge in the bulk to its horizon is only invertible if one knows the law of characteristic propagation from the horizon into the bulk; in the interaction free case this means the knowledge of the bulk mass which was lost in the holographic projection. This law has no geometric presentation, i.e. the local substructure of a wedge algebra $\mathcal{A}(W)$ cannot be geometrically encoded into $\mathcal{A}(H(W))$, although the two global algebras are identical. This also applies to event horizons in curved spacetime; is incompatible with the idea that the *full* information contained in the local bulk substructure of a region can be locally encoded into its horizon.

In discussing the horizon-bulk relation it is easy to overlook the fact that the representation of the lightfront generators in terms of the Wigner creation

to work by Kay and Wald from the 90s [24]. A presentation of free field holography from a more functional analytic point of view can be found in [33] and references therein.

and annihilation operators $a(p), a^*(p)$ as in the first line of (21) is not intrinsic, rather the intrinsic characterization of the theory is contained in the structure of it correlation functions or its commutation relation. The characteristic data in the interaction free case have lost all reference to a mass and unless one adds this information there will be no unique holographic inversion. The uniqueness situation is in no way better in interacting theories. Even in the free field situation the mutual fuzziness between compact localized regions on $H(W)$ and regions in the W-bulk or the inverse situation remains. In the spirit of LQP intrinsicness, the reconstruction of the local substructure $\mathcal{A}(W)$ requires the knowledge of the action of the Poincare group or that of the relative positions of several null-surfaces. It turns out that the enlargement of group symmetry beyond the 7-parametric subgroup and the increase of degrees of freedom through the relative positioning are alternative ways for reconstructing the bulk from it holographic projection (more remarks below).

Not knowing anything about QG, it is difficult to refute or support the claim that there are holographic "screens" in QG which store *all* bulk information; but this is definitely not the situation in QFT on causal horizons or on event horizons as they occur in QFT in CST. There are fewer degrees of freedom in a QFT on LF than in the bulk QFT. More knowledge as that of the action of LF-changing Poincaré transformations increases the cardinality of degrees of freedom. The usefulness of the holographic LF projection is inexorably linked to the thinning out of degrees of freedom. The best way to appreciate this happy circumstance is to look at correspondences for which this fails to be true (see below).

Attempts at nonperturbative constructions of QFT inevitably amount to subdividing the problem into simpler pieces which only address certain aspects of the holistic QFT project. Holography on horizons is a radical spacetime reordering of a given quantum matter substrate. The latter may be a Weyl algebra (the more rigorous formulation for (20) and (21)), a CAR algebra, or that described in terms of a local interaction. In all cases the spacetime reorganization according to the LF ordering structure simplifies certain field aspects at the expense of particle aspects which become masked (hidden in the holographic inversion for which more information is required).

Behind this idea of "thinning" degrees of freedom and loosing information in holographic projection there is the concept of a *natural phase space density* for a given spacetime dimension. Intuitively speaking the idea behind this is to hold onto as many properties as possible from Lagrangian QFTs in situations outside the Lagrangian quantization setting. As mentioned in section 2 of the part I this notion, introduced by Haag and Swieca [5] in the 60s and later refined to the nuclearity requirement by Buchholz and Wichmann [6] in the 80s, demands that the phase space density of degrees of freedom in QFT, which is compatible with modular localization, is bigger than the *finite* degrees of freedom per phase space cell of QM; but the infinite degrees of freedom also should not go beyond that of a *nuclear* set, since otherwise the causal propagation, the existence finite temperature statistical mechanics and the asymptotic particle interpretation will be endangered.

Such a situation arises in the AdS_5-CFT_4 correspondence because if one choses one side, say the one with the larger spacetime dimension, as being of Lagrangian origin (i.e. with a natural phase space density), the other side of a correpondence is uniquely determined and the only thing one can do is to look whether its degrees of freedom are natural or not. The naive argument would suggest that when one passes to a lower dimensional world one has too many degrees of freedom i.e. naturality is lost. In the opposite direction one expects that the 5-dimensional AdS theory obtained from a natural CFT_4 model is too "anemic" the AdS theory coming from a normal CFT turns out to be "anemic". Both statements can be made precise and exemplified by explicit free field calculations starting from either side [8].

Strangely enough, although noticed very preciesly by Rehren, who gave a mathematical proof of this correspondence[12] [26][27], this issue has not been addressed by the Maldacena community [28] who first formulated the conjecture about the correspondence in the context of a conjecture concerning the relation of gravity on AdS_5 with a conformal supersymmetric N=4 Yang-Mills theory in 4 dimensions both thought of as theories with standard physical degrees of freedom. There is a vast community with more than 6000 publications who tried to support Maldacena's conjecture, but scientific truth are not decided according to the size of globalized communities. In fact such communities follow completely different pattern[13] than a critical discourse between individuals or small groups of individuals collaborating on one subject. The conjecture has soon its 20^{th} anniversary with no tangible result but an ever larger number of publications with increasingly outragious claims.

The fact is, and every particle physicist of a sufficient age will confirm this, that several decades of community building around the idea of a "theory of everything" and extremely bad leadership has created an expectation of salvation in which the level of knowledge falls far back behind what is needed for research at the frontiers of QFT.

Although not so obvious, the degrees of freedom argument can also be applied to brane physics; against naive intuition branes contains the same cardinality of degrees of freedom as the bulk[44] and hence the same arguments about spacetime dimenion compatible naturalness applies.

We have seen that the holography of bulk matter on W to the horizon $H(W)$ is not a correspondence but a projection. So it is clear that the loss of information or the reduction of degrees of freedom for the preservation of naturalness is a priviledge of holography on null-surfaces. This explains why holography onto horizons is extremely useful. For the case at hand, namely the bulk- and lightfront- generators, this projective nature of holography asserts itself via the fact one cannot reconstruct the bulk from the space of $H(W)$. But

[12] Something which is ill in the physical-conceptual setting, maybe perfect on the mathematical side.

[13] In fact for the first time in the history of particle theory there is a deep schism between a majority who has been raised in the shadow of a theory of everything and a scholarly minority with profound knowledge of QFT who are in a ivory tower against their own choice. Particle theory has entered a deep crisis.

the holographic projection is nevertheless very useful because it contains still a lot of informations about the bulk in a much simpler more accessible fashion. It is this aspect of simplification at the expense of information completeness which makes holographic projection that useful. Of course this could also happen in the case of correspondences; even though a CFT_4 viewed from the unphysical AdS_5 description has lost its physical interpretation, certain mathematical aspects may still simplify.

Historically the "lightcone quantization" which preceded lightfront holography shares with the latter part of the motivation, namely the idea that by using lightlike directions one can simplify certain aspects of an interacting QFT. But as the terminology "quantization" reveals that this was mixed up with the erroneous idea that in order to achieve simplification one needs a new quantization instead of a radical spacetime reordering of a given abstract algebraic operator substrate whose Hilbert space is always maintained. As often such views about QFT results from an insufficient appreciation of the autonomy of the causal locality principle by not separating it sufficiently from the contingency of individual pointlike fields.

Formally mass shell representations also exist for interacting fields. In fact they appeared shortly after the formulation of LSZ scattering theory and they were introduced in a paper by Glaser, Lehmann and Zimmermann [29] and became known under their short name of "GLZ representations". They express the interacting Heisenberg field as a power series in incoming (outgoing) free fields. In case there is only one type of particles one has:

$$A(x) = \sum \frac{1}{n!} \int_{V_m} \cdots \int a(p_1, ...p_n) e^{i\sum p_k x} : A_{in}(p_1)...A_{in}(p_n) : \frac{d^3p_1}{2p_{10}} ... \frac{d^3p_1}{2p_{10}} \quad (22)$$

$$\begin{aligned} &A_{in}(p) = a_{in}^*(p) \ on \ V_m^+ \ and \ a_{in}(p) \ on \ V_m^- \\ &a(p_1, ...p_n)_{p_i \in V_m^+} = \langle \Omega | A(0) | p_1, ...p_n \rangle \end{aligned} \quad (23)$$

where the integration extends over the forward and backward mass shell $V_m^{\pm} \subset V_m$ and the product is Wick ordered. The coefficient functions for all momenta on the forward mass shell V_m^+ are the vacuum polarization components of A and the various formfactors (matrix elements between in "ket" and out "bra" states). In the GLZ setting the coefficient functions arise as the mass shell boundary values of Fourier-transformed retarded functions.

The convergence status of these series is unknown[14]. This mass-shell representation is inherently nonlocal. Nevertheless one may hope that it does not only represent a local bulk field but that its light front restriction is also local. Superficially there is no problem with placing the GLZ representation on the lightfront. However the application to $:A^2(x):$ the Wick-ordered composite of the free field shows that there is an obstruction against a simpleminded pointlike formulation (18) since the Wick decomposition of $:A^2(x):_{LF}:A^2(x'):_{LF}$ contains squares of transverse delta functions which, as the result of

[14] In contrast to the perturbative expansion which is known to diverge even in the Borel sense, the convergence status of GLZ had not been settled.

having lost the energy momentum positivity in the transverse components, are incurably divergent. This *transverse delta problem* is absent in the holographic projection of two-dimensional massive theories. The decisive property is however not whether generating fields on LF come from pointwise manipulations on bulk fields, but rather whether a net on LF can be described of generating fields. But since the concrete calculations in terms of individual fields is more familiar one would like to hope that there is a solution to the transverse delta problem.

The holography on horizons contains some not entirely understood problems of spin and statistics. Only Bose fields with integer short distance dimensions, as those associated with conserved currents (conserved charge currents, the energy-momentum tensor), can have bosonic holographic projections whereas (bosonic or fermionic) bulk field with anomalous short distance dimensions pass to plektonic lightfront fields for which the anomalous dimension, the anomalous spin and their braid group statistics are interconnected via the chiral spin&statistics theorem [10]. This change of the statistics in passing from bosons/fermions with anomalous dimensions to lightfront fields with anyonic/plektonic statistics is formally taken care of by the GLZ formula.

These transmutation properties with respect to statistics are more conveniently studied in the simpler context of absence of transverse dimension i.e. in the holographic projection of two-dimensional QFTs onto the lightray. In this case the aforementioned obstruction is absent. In particular for the factorizing models presented in the section on algebraic aspects of modular theory, there are on-shell representation of local fields in terms of certain wedge generating creation/annihilation operators, the Zamolodchikov-Faddeev algebra generators (see part I), which replace the incoming creation/annihilation operators in (22) and lead to a coefficient functions which are identical to the crossing symmetric formfactors.

The pointlike fields in the mass shell representation highlight some interesting problems whose better understanding is important for autonomous nonperturbative constructions of models in QFT i.e. constructions which do not depend on Lagrangian quantization as those presented in part I. The more rigorous algebraic method by its very nature (using relative commutants) only leads to bosonic holographic projections. This means that the extended chiral structure on the lightfront only contains integral values in its short distance spectrum; i.e. the generating fields are of the kind of the chiral components of two-dimensional conserved currents and energy-momentum tensors. Hence only a small subalgebra of the bulk algebra[15] associated with transverse extended currents, energy momentum tensor etc. will have a bosonic holographic image; there would be no anomalous dimension field in the algebraic holographic projection. Apart from conserved currents whose charges must be dimensionless, fields are not protected against carrying non-integer short distance scale dimensions; such fields would not pass the algebraic method of holography.

[15] Apart from conserved currents whose charges must be dimensionless, fields are not protected against carrying non-integer short distance scale dimensions.

Clearly some of these ideas, as important for the future development of QFT as they may appear, are not yet mature in the sense of mathematical physics. Therefore it is good to know that there exists an excellent theoretical laboratory to test such ideas in a better controlled mathematical setting, the two-dimensional factorizing models and their this time bona fide (no transverse extension) chiral holographic projection. From a previous section on modular theory in part I one knows that these models have rather simple on-shell wedge generators $Z(x)$ which still maintain a lot of similarity with free fields. In that case Zamolodchikov proposed a consistency argument which led to interesting constructive conjectures about relations between factorizing models and their critical universality classes represented in form of their conformal short distance limits.

From a conceptual viewpoint the critical conformal limit leading to universality classes is very different from the holographic projection. The former is a different theory whose Hilbert space has to be reconstructed from the massless correlation function, whereas the latter keeps the original Hilbert space and only reprocesses the spacetime ordering of the original quantum substrate. Assuming that one knows the chiral fields on the lightray as a power series in term of the Zamolodchikov-Faddeev operators[16] [30], one has a unique inversion, i.e. the holographic projection becomes an isomorphism.

Calculations on two models [31], the Ising field and the Sinh-Gordon field, have shown that the universality class method and the holographic projection lead to identical results[17]. Whereas the anomalous dimension of the Sinh-Gordon field can only be computed approximately in terms of doing the integrals in the lowest terms in the mass shell contributions, the series for the Ising order field can be summed exactly and yields the expected number 1/16. This is highly suggestive for reinterpreting the more speculative Zamolodchikov way of relating factorizing models with chiral models in the conceptually clearer setting of holographic projections.

The gain in modular generated symmetry is perhaps the most intriguing aspect of holography. In general the modular theory for causally complete spacetime regions smaller than wedges leads to algebraic modular groups which cannot be encoded into diffeomeophisms of the underlying spacetime manifold; the generators of these groups are at best pseudo-differential operators. However there are strong indications that their restriction to the horizon are always geometric. So it may be useful to construct the bulk modular groups from those of their holographic projection. The constructive knowledge about chiral theories has very much progressed [32] and it would be nice to be able to use that insight to construct massive bulk theories with chiral models being the holographic input.

Let us finally address the symmetry enhancement which leads to the infinite Bondi-Metzner-Sachs symmetry group which these authors discovered in

[16] From the point of view of chiral models such a representation is of course somewhat unusual.

[17] The consistency of the holographic lightray projection with the critical limit for factorizing models was verified in s blackboard discussion with Michael Karowski.

asymptotically flat solutions of classical general relativity. In the case of the free field it is not difficult to see [22] that the absence of transverse vacuum polarization leads to a slightly larger symmetry than the transverse Euclidean group; the transverse delta functions permits a compactification to the Riemann sphere on whose complex $\zeta, \bar{\zeta}$ coordinates ($\zeta = x + iy$) the group SL(2,C) acts as a fractional transformation, just as the covering of the Lorentz group. Restricting the Diff(S^1) group to the symmetry group of the vacuum which is the finite parametric Moebius group; imposing in addition the requirement of the preservation of the point at infinity in the lightlike direction the group is the ax+b translation dilation group. By itself this would be a two parametric group, but the fact that the two parameters can be functions of $\zeta, \acute{\zeta}$ makes jointly generated group an infinite parameter group

$$
\begin{aligned}
x &\rightarrow F_\Lambda(\zeta, \bar{\zeta})(x + b())\zeta, \bar{\zeta} \\
(\zeta, \bar{\zeta}) &\rightarrow U(\Lambda)(\zeta, \bar{\zeta}), \; U(\Lambda) \in SL(2, C)
\end{aligned}
\tag{24}
$$

The group composition law $F_{\Lambda'}(\Lambda(z, \bar{z}))F_\Lambda(z, \bar{z}) = F_{\Lambda'\Lambda}(z, \bar{z})$ requires the multiplicative factor to be of the form

$$
F_\Lambda(\zeta, \bar{\zeta}) = \frac{1 + |\zeta|^2}{|a\zeta + b|^2 + |c\zeta + d|^2} \tag{25}
$$

whereas the functions $b(\zeta, \bar{\zeta})$ are from a function space which is the closure of $C^\infty(\zeta, \bar{\zeta})$ functions on the Riemann sphere in some topology. The somewhat unexpected property is that the action of SL(2,C) on the function space contains (in its linear part) the a copy of the semidirect product action of the Lorentz group on the translations i.e. the infinite dimensional BMS group contains the Poincare group. For more informations especially on the position of the Poincare inside the BMS group we refer to a comprehensive paper by Dappiaggi [33].

One expects this transformation on a classical Penrose double cone horizon at infinity since on such a "screen" the Poincaré group acts naturally. But its appearance already on compact quantum double cones is at first sight somewhat astonishing although the split property yields a mathematical explanation [22].

It is helpful to take notice that in addition to the thermal property of the vacuum reduced to one tensor factor as explained in the previous section, the split property permits also to "localize" global symmetries which constitutes an analog of the classical Noether theorem [2]. This pure algebraic derivation does not require to define a conserved current with the help of the Lagrangian quantization, one even does not have to know how to construct without being forced to postulate the existence of singular current operators as the quantum counterparts of the classical conserved Noether currents. This intrinsic (i.e. not relying on a quantization parallelism) "localization" of global symmetries based on the split property also applies to the localization changing Poincaré symmetry if one restrict the group parameters to sufficiently small values so that the localization of the transformed operators stays inside the chosen localization region [2].

Using the notation of the double cone localization defined in the previous section one obtains a representation of the full Poincaré group on the tensor factor $\mathcal{N}$ which for sufficiently small parameters act on operators $A \in \mathcal{A}(\mathcal{D}(R))$ the same way as the global symmetry. In the ring region or its light sheet prolongation $\mathcal{D}(R)\backslash D((R+\Delta R))$, which constitutes the fuzzy localized part of $\mathcal{N}$ which surrounds its sharply localized nucleus $\mathcal{A}(\mathcal{D}(R))$, the Poincare group does not act geometrically in a way which can be encoded into a geometric diffeomorphism; of course it never fails to be an algebraic automorphism. Hence the split situation for a double cone creates an analog situation to a Penrose screen except that the Poincaré subgroup of the BMS group is an unphysical extension of the partial physical Poincaré group for parameter values for which the boundary of the region of interest in $\mathcal{A}(\mathcal{D}(R))$ passes into the ΔR split ring-like or light-sheet region with $\Delta R \to 0$ in the holographic limit in which *light-sheet $\to$ holographic screen*. With other words the artifact a Poincaré subgroup of the holographic BMS group is explained in terms of the artifact of a localized Poincaré symmetry resulting from the split construction.

6 The local covariance principle

Less than a decade ago the holistic structure QFT in CST was significantly enriched by the formulation of the local covariance principle [39]. Preliminary studies in this direction began at the beginning of the 90s with the realization that even in the case of a free quantum field the definition of an energy-stress tensor with properties similar to those of the classical expression which enters the right hand side of the Einstein Hilbert equation is a very nontrivial matter as soon as curvature enters [37]. One problem is that even in Minkowski QFT, where a unique definition in terms of the Wick-ordered expression of the classical form is available, the energy density is not bounded below, since one can find state vectors on which the energy density $T_{00}(x)$ takes on arbitrarily large negative values [34]. Whereas this result does not create serious problems in standard QFT, it causes problems with the quantum counterpart of certain stability theorems which follow from positivity inequalities for the classical stress energy tensor which enters on the right hand side of the Einstein Hilbert equation.

It started a flurry of investigations which led to state-independent lower bounds of $T_{00}(f)$ for fixed test functions as well as inequalities on subspaces of test functions [35]. These inequalities which involve the free stress-energy tensor were then generalized to curved space time[18]. In the presence of curvature the main problem is that the correct definition of $T_{\mu\nu}(x)$ is not obvious since in a generic spacetime there is no vacuum like state (which is distinguished by its high symmetry) to which the operator ordering could refer; to play that point split game with an arbitrarily chosen state will not produce a locally covariant energy stress tensor since states (in contradistinction to operators) are inevitably global in that their dependence on the spacetime metric is not

[18]For recent publication with many references see [36].

limited to the infinitesimal surrounding of a point (which would be required by a local covariance principle).

A strategy to obtain locally covariant local quantum field product for the energy-momentum tensor which is not associated with a particular state was given in 1994 by Wald [37] in the setting of free fields. His postulates gave rise to what is nowadays referred to as the *local covariance principle* which is a very nontrivial implementation of Einstein's classical covariance principle of GR to quantum matter in curved spacetime (after freeing the classical principle from the relics of its physically empty coordinate invariance interpretation). The requirements introduced by Wald determines the correct energy-momentum tensor up to local curvature terms (whose degree depends on the spin of the free fields). The method of Wald is somewhat surprising since it does not consists in taking the coincidence limit after subtracting from the point split expression the expectation value in one of the states of the theory. Rather one needs to subtract a "Hadamard parametrix" [38] i.e. a function which depends on a pair of coordinates and is defined in geometric terms; in the limit of coalescence it depends only on the metric in a neighborhood of the point of coalescence. Only then the global dependence on the metric carried by states can be eliminated in favor of a local covariant dependence on $g_{\mu\nu}(x)$ and its derivatives. As a result the so-constructed stress-energy tensor at the point x depends only on the metric in an infinitesimal neighborhood of x.

Already Wald's work contains the important message that in order to construct the correct tensor it is not enough to look at one model of a QFT in a particular curved spacetime background, but one is obliged to look *simultaneously at all different spacetime orderings of abstract quantum matter* (in Wald's case the abstract Weyl algebra quantum matter) in order to be able to correctly describe the algebraic structure of one particular model. The implementation of the local covariance principle requires a strict separation of the algebraic structure from states; settings of QFT in which the two are mixed together as functional integral approaches or other formulations in terms of expectation values are unsuitable. In fact it is not an exaggeration to think that without the dichotomy between spacetime indexed nets of operator algebras and states inherent in algebraic QFT, the formulation of QFT in CST would not reached the present level of clarity.

In [39] the formulation of the local covariance principle attained its present form. There are two different but connected formulations, one working with nets of causally closed nets of spacetime-indexed operator algebras and the other one in terms of pointlike covariant fields. The difference to the standard formulation of a global algebra with its causally closed subalgebras is that algebras which are "living" on isometric causally closed parts of spacetime and are in addition algebraic isomorph (are made from the same abstract matter substrate) are considered on equal footing. The totality of observation which can be made on isometric isomorphic subalgebras is identical and independent of differences which may show up in their surrounding. This goes a long way towards what is considered as the characterizing property of QG: the background independence. Some researchers of QG want to go one step beyond isomorphy and look for

equality in the spirit of gauge invariance by integrating over gauge fields but a proposal to implement this ides is still missing.

The local covariance principle can also be expressed in terms of pointlike covariant (under local isometries) fields. In contradistinction to standard spacetime symmetries in QFT (e.g. Poincaré symmetry) these symmetries do not come with a state which is left globally invariant. They are like the Diff(S^1) symmetry beyond the Moebius group of chiral conformal QFT on the circle in which case there is also no state which is globally invariant under diffeomorphism beyond the Moebius group.

Recently these renormalization ideas were applied to computations of backreactions of a scalar massive free quantum field in a spatially flat Robertson-Walker model [41]. As a substitute for a vacuum state one uses a state of the Hadamard form since these states fulfill a the so-called microlocal spectrum condition which emulates the spectrum condition in Minkowski spacetime. The singular part of a Hadamard state is determined by the geometry of spacetime. The renormalization requirements of Wald lead to a an energy momentum tensor with 2 free parameters which can be conveniently represented as functional derivatives with respect to the metric of the two quadratic invariants which one can form from the Ricci tensor and its trace. In [40] the resulting background equations were analyzed in the simpler conformal limit and it was found that the quantum backreaction stabilizes solutions i.e. accomplishes a task which usually is ascribed to the phenomenological cosmological constant. Without the simplifying assumption the linear dependence on a free renormalization parameter guaranties that any measured value can be fitted to this backreaction computation. The principles of QFT cannot determine renormalization parameters.

Hence from a QFT point of view there is no cosmological problem which places QFT in contradiction with astrophysical observations. A consistency check would only be possible if there are other measurable astrophysical quantities which fall into the setting of quantum backreaction on spatially flat RW cosmologies.

Last not least the requirement of the local covariance principle to consider a given quantum matter substrate simultaneously in *all* CST helps to maintain some aspects of particles, whose Wigner characterization only applies to Minkowski spacetime. Since the latter is included in the covariance definition it is sufficient to find an region of the given CST which is isometric to a Minkowski space region in order to secure objects which behave in a certain limited spacetime region as particles (see last section).

7 Resumé, miscellaneous comments and outlook

For a long time the conceptual differences between relativistic quantum mechanics[19] and QFT in which the maximality of propagation is build into the

[19]In relativistic quantum mechanics (DPI of part I) the velocity of light is not a maximal propagation over finite distances but rather a limiting velocity for the leading asymptotic

algebraic causality structure were not sufficiently appreciated. Even in contemporary articles one finds the terminology "relativistic QM" instead of QFT. Perhaps one reason is that many people believe that relativistic QM, as a separate subject from QFT, does not really exist so that the somewhat sloppy terminology does not really matter. But the existence of the DPI presented in part I shows that this is not correct; the direct particle interaction theory is a relativistic theory of particles which fulfills all requirement which one is able to implement using exclusively properties of particles. As mentioned in part I, even creation/annihilation processes of particles in scattering processes can be described in DPI by introducing suitable channel couplings "by hand". What is however characteristic of interacting QFT and has no place in DPI is the notion of interaction-caused infinite vacuum polarization. In part I the fundamental differences were explained in terms of two fundamentally different localization concepts.

Fortunately these very different localizations coalesce asymptotically i.e. the quantum mechanical Born-Newton-Wigner localization becomes covariant in the asymptotic limit of scattering theory and its quantum mechanical probability concept permits to extract cross sections from scattering amplitudes. So perhaps it is better to de-emphasize the "bottle half-empty" view expressed in (see part I) the title *Reeh-Schlieder defeats Born-Newton-Wigner* and take a more harmonic perspective by viewing *R-S and BNW, as an asymptotically harmonious pair.* Any other result would have caused a disaster in the relation between particles and fields. DPI reaches its conceptual limit if it comes to the notion of formfactors.

It is an interesting question whether LQP has any new message for the main philosophical problem of the 20th century posed by QT: the controversy between Bohr's (and more generally the Copenhagen) *holistic view of quantum reality* and Einstein's *insistence in independent elements of reality.* I think it does. On the one hand it pushes the holistic point of view to its extreme as exemplified in the various ways it realizes an extreme form of connectedness which we tried to highlight by calling it "Murphy's theorem" ("what can couple does couple") as illustrated by Reeh-Schlieder property, the analytic crossing connection of the different formfactors of a local operator with its vacuum polarization and in a much stronger form by the characterization of a LQP in terms of a finite number of monads in a specific modular position. But on the other hand there is also the split property which creates a situation close to Einstein's view. If one interprets Einstein's maxim in an appropriate way, namely as the preservation of the totality of all possible measurements in the presence of uncontrolled activities in a spacelike separated laboratory instead of excluding the holistic EPR situation, then there is no antagonism between the two views. The reconciliation maybe difficult from an intuitive viewpoint, but the existence of a mathematical consistent presentation clearly shows that intuition is not always reliable and sometimes needs mathematical guidance.

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contribution of a wave funtion. In this respect it is the relativistic counterpart of the speed of sound in a nonrelativistic system of coupled oscillators.

The particle-field relation in the presence of interactions is one of the subtlest aspect of relativistic local quantum physics; there has never been any closure on this issue, nor would anybody who has a detailed knowledge about this subject expect one in the near future. Nobody at a high energy laboratory has ever directly measured a hadronic quantum field[20]. Even though all our intuition and the formulation of principles enters the theory through local fields and spacetime indexed algebras of observables generated by them, quantum fields remain hidden to direct observations. What one really measures are either particles entering and leaving an interaction process, or thermal radiation densities and their fluctuations as in the microwave background radiation. Quantum fields or local observable algebras are the carriers of the causal locality principle[21] but, different from classical relativistic fields which propagate with a maximum velocity, they have themselves no ontological status. The protagonists of LSZ scattering theory coined a very appropriate word for this state of affairs, they called fields in particle physics "interpolating". In general there will be infinitely many interpolating fields which interpolate the same particle. But there are indications based on the use of the crossing property, that the inverse scattering problem has a unique solution with respect to the system of local algebras [42] without any guaranty for its existence.

Besides the standard Wigner particle setting whose connection with fields is channeled through the (LSZ, Haag-Ruelle) scattering theory there are charged (infra)particles whose scattering theory in terms of inclusive cross sections exists in the form of computational recipes without conceptual backup. These particle-like objects correspond to charged fields in QED which only exist as semiinfinite strings i.e. are nonlocal (in the standard use of this word where everything which is not pointlike generated is called nonlocal. It would be naive to expect that the situation with respect to the necessity of introducing physical nonlocal observables decreases in passing from abelian gauge theories to Yang-Mills theories.

But it is precisely the idea of an equivalence class of interpolating local fields in their property of interpolating the same particle which led to the powerful observed properties as e.g. Kramers-Kronig dispersion relations which a particle-based approach as DPI can not deliver. The experimental verification of a dispersion relation cannot select or rule out a particular Lagrangian model of hadronic interactions but rather is a test for the validity of the causal localization principle.

The particle based view is certainly thrown into disarray when one studies QFT in non inertial frames (e.g. the Rindler frame of the Unruh effect) or in CST. According to the best of my knowledge there exists no time-dependent LSZ scattering in a (flat) Rindler world; although the global and the wedge-localized QFT live in the same Hilbert space, the global particle states carry no intrinsic

[20]There are however certain distinguished composite fields, in particular the quantum analogs of Noether currents, whose formfactors are used in the analysis of scattering data for certain deep inelastic processes.

[21]In the noninteracting case covered by Wigner's representation theory this viewpoint has led to the understanding of string-localized generators of "infinite spin" representations [7].

physical information with respect to the wedge-localized theory. Robert Wald, a leading researcher on QFT in CST, has recently proposed [38] to abandon the particle concept altogether and work under the hypothesis that fields are directly measurable. But measurability requires a certain amount of stability and individuality; quantum fields are fleeting objects of which there are always infinitely many for which, in contradistinction to classical fields, there seems to exist no measurable property which allows to distinguish the members in an equivalence class of fields which carry the same charge. It is hard to believe how Wald's advise of abandoning particles could work. Perhaps, as indicated at the end of the previous section, the local covariance law leads to an argument why certain particle manifestations in Minkowski spacetime can be transfered to finite regions in CST.

In recognition of this lack of observational distinctness for fields, the algebraic approach to QFT has placed *spacetime-indexed operator algebras* into the center stage. In such a setting the increase of knowledge about a localized operator algebra takes place through a tightening in localization and not via the increase in precision in measuring properties of an individual operator[22]. This fits very nicely with scattering theory because the in/out fields resulting from different operators in the same algebra $\mathcal{A}(\mathcal{O})$ are identical; their differences become absorbed into normalization factors [43] and it is at best the system of operator algebras which is determined by inverse scattering and never the individual fields.

Although without the notion of particles and scattering theory the physical world of QFT in CST would be quite a bit poorer, it is by no means void of all experimentally accessible aspects. Even if one has no clear idea on the nature of the cosmological reference state of our universe (the CST replacement for the vacuum), one can study models and compare the thermal aspects of the expectation values of the energy-stress tensor in the cosmic reference state with data from the cosmic background radiation [40], for this one does not need the vacuum state and particle states as they follow from Poincaré symmetry.

A class of objects between particles (on-shell) and fields (off-shell) which are ideally suited for the study of vacuum polarization are the *formfactors*, i.e. matrix-elements of local operators between *bra* out- and *ket* in- particle states. The special matrix-elements with vacuum on one side and all particles on the other side characterize the vacuum polarization of the local "bang" on the vacuum $A\Omega,\ A \in \mathcal{A}(\mathcal{O})$. The general formfactor results from the vacuum polarization component by a particular on-shell analytic continuation process known as the *crossing property.* The latter is one of the most subtle property in the particle-field relation [44], its comprehension goes significantly beyond that of time-dependent scattering theory. As the Unruh effect, it uses KMS properties of the wedge localized algebra[23], the subtle point being that one needs to construct very special wedge localized operators which applied to the vacuum

[22] An exception are those localized individual operators which result from the "split localization" of global symmetries (the before mentioned quantum Noether currents).

[23] It testifies to the conceptual depth of modular localization that it places such diverse looking issues as the Unruh effect and the crossing property under one roof.

generate particle states without admixture of vacuum polarization cloud [44]. The history of the crossing property is also a prime example of the disastrous consequences of a several decades lasting misunderstanding of a central concept of QFT [44] in the absence of a profound criticism.

An example of a deep antagonism of QFT with respect to QM which is usually not perceived as such comes from the exploration of modular localization. Whereas the localization of states is basically a kinematical notion, its algebraic version incorporates most, if not all dynamics. The crucial property is the monad (hyperfinite type III_1 factor) nature of the local algebras. In QM all Born-localized subalgebras are of the same type as the global algebra, namely type I_∞ factor $B(H)$, $H \subset H_{glob}$. A monad in QM only appears at finite temperature in the thermodynamic limit. There is hardly any textbook which emphasises the radically different algebraic properties (see however [45]) from those of its "boxed" Gibbs state approximands[24].

In QFT as opposed to QM, it is the monad structure which is the normal situation and the quantum mechanical type I_∞ property which is the exception; the latter can only be constructed by "splitting" a local algebra from its causal disjoint and in this way creating a fuzzy "halo" in which the vacuum polarization can settle down to a (halo-dependent) temperate behavior leading to a finite (halo-dependent) entropy. So the region for the calculation of entropy is not the horizon itself but rather a light-sheet surrounding the horizon of the localization region. Hence the divergence of localization entropy in the limit of vanishing sheet size $\Delta R \to 0$ is not an indication that QG must intervene in order to rescue QFT from high energy inconsistencies [16], but rather that the assumption of tensor factorization, which is the prerequisite of a bipartite entanglement situation, was not quite correct; the total algebra $B(H) = \mathcal{A} \vee \mathcal{A}'$ cannot be written as a tensor product even though $\mathcal{A}^{'}$ is the commutant of $\mathcal{A}$.

The split construction enforces the tensor product situation but it brings a new parameter into the fray, the split size ΔR. The conceptual situation calls for great care in using standard notions of quantum information theory from QM in quantum field theoretical situations in which thermal aspects of entanglement (and not the information theoretical) are dominant. In particular the discussions about information loss in black hole physics seems to have been carried out without much appreciation for the field theoretical subtleties addressed in this essay. Although the terminology "entanglement" strictly speaking does not apply to a bipartite separation with sharp causal boundaries in QFT, the literature on entanglement unfortunately does not seem to differentiate between the QM and the QFT case. There is of course the problem of respecting a historically accepted terminology when its literal meaning contradicts mathematical facts [46].

Another issue presented in part I is the question to what extend one needs

[24]The thermofield formalism of doubling of degrees of freedom holds for the finite box and corresponds to the tensor factorization between the boxed algebra and its commutant. But by not noting that this factorization breaks down in the thermodynamic limit the aficionados of thermofield theory miss an interesting chance of becoming aware of a deep conceptual problem.

to go beyond pointlike generators. We reviewed the Wigner representation theory in the modular localization setting and reminded the reader that the only class which needs stringlike generating covariant wave function is Wigner's infinite spin class which after a more than 60 year odyssey, thanks to modular localization, finally reached its final position with respect to localization. The algebraic notion of stringlike generator of an algebra is however more restrictive in the sense that it is described by an "indecompoaible" string-like localized field $\Phi(x,e)$ (with e the spacelike direction of the semiinfinite string and x its start) i.e. one which cannot be resolved in terms of a pointlike field smeared along $x+\mathbb{R}_{+}e$. The application of such an algebraic string to the vacuum state is however decomposable as a state into irreducible representations of the Poincaré group and unless there are infinite spin components, the localization of the state is pointlike even though the algebraic object was an indecomposable string[25]. The only illustration for such an algebraic string, mentioned in the first part, is the Dirac-Jordan-Mandelstam string. The Buchholz-Fredenhagen setting [47] offers room for pure massive strings as one wants them in QCD, but since these strings do not leave any traces in perturbation theory, they remain beyond what one is able to control with existing methods. We also stressed that the widening of the setting of localization achieved through the modular formalism poses new questions involving massive vectormesons whose resolution could be relevant for the interpretation of forthcoming LHC experiments.

Localization is the overriding principle of LQP, in fact it is the only principle and therefore the main and often difficult task in the conceptional conquest of specific effects and mechanisms in QFT consists in figuring out how and under what conditions they can be derived from localization. In phenomena first observed in a Lagrangian quantization setting as e.g. QED infrared properties, spontaneous symmetry-breaking a la Goldstone, the Schwinger-Higgs screening mechanism or the crossing property most of the rich conceptual-mathematical understanding came from the pursuit of this goal, and if there has not yet been a perfect understanding, it only means the the subtle connection of these phenomena with localization has not yet been completely unraveled. The story of the Wigner infinite spin representation class shows that even for kinematical problems the understanding of their localization aspects sometimes take more than half a century (see part I).

Even where it is least expected, namely in case of the mysterious quantum concept of *internal symmetries* is a particular mode of realization of the locality principle. The DHR theory showed how the possible superselected representations of an observable algebra[26] and the ensuing group theory which ties the different sectors into one "field representation" (on which it acts in such a way that the local observable algebra reemerges as the fix point subalgebra under

[25] The state localization structure is exclusively determind by the representation theory of the Poincaré group whereas the problem of irreducible algebraic generators in interacting theories depends on the dynamics.

[26] This example (the Doplicher-Haag Roberts superselection theory [2]) is particularily suitable since nobody would expect group theory to emerge from classifying inequivalent local representations of the observable algebra; at no point is group theory visible.

the action of a compact group) is uniquely contained in the structure of the local observables. On the other hand the monad picture shows that even spacetime symmetries can be encoded in the abstract modular positioning within a shared Hilbert space.

A brief explanation of this last remark is as follows (see part I). A QFT in the algebraic setting is a net of spacetime-indexed algebras. Hence it comes as somewhat of a surprise that one can do with less; to get a concrete QFT going, one only needs a finite number of monads in a special relative modular position. The reason why I used a whole section in part I of may essay for a description of this property (which up to now has remained without practical use), is that I find this very exciting from a philosophical point of view. It is the almost literal adaptation of Leibniz's idea of what constitutes reality to the setting of local quantum physics. A monad in isolation is not much more than a point in geometry, besides the absence of pure and mixed states and the statement about what kind of states it admits instead it, is an object without properties. Surprisingly its is not even necessary to require that the algebras are monads, their modular theory together with the positioning defined in terms of modular inclusions or modular intersections [48] alone forces the factors to be of hyperfinite type $\mathrm{III}_1,$ no other factor algebras can be brought into that particular modular position. The structural richness of QFT result solely from the relation between these monads; this includes not only the local net of quantum matter but also its internal as well as spacetime symmetries. There are other equivalent ways to characterize a QFT in terms of the modular data of its local subalgebras [51]. From a practical point of view it turns out to be more useful to know the action of the Poincare group on the generators on one fixed wedge which is equivalent to the positioning point of view, indeed this was the approach by which the existence of certain two-dimensional factorizing models was established (section 5 of part I).

Perhaps the most profound conceptual-philosophical contrast between QM and QFT finds its expression in these modular encoding. As mentioned in the last section in part I, there have been other ideas which are designed to highlight an underlying relational nature of QT; in particular Mermin's view of QM [50] in terms of its *correlations*. It is difficult to find a mathematical backup which is as crystal clear as that in terms of modular positioning of monads. Mermin expressed his relational point of view by the following apodiction: *correlations have physical reality, that what they correlate does not.*

The LQP analog of this dictum would be: *relative modular positions in Hilbert space have physical reality, the substrate*[27] *which is being positioned does not.*

The presentation of QFT in terms of positioning monads is very specific of LOP i.e. it has no analog in QM i.e. Mermin's relational view is not a special case of positioning in LQP. It has the additional advantage that beyond the

[27]Modular positioning is the most radical form of relationalism since the local quantum matter arises together with internal and spacetime symmetries. In other words the concrete spacetime ordering is preempted in the abstract modular positioning of the monads in the joint Hilbert space.

metaphor there are hard mathematical facts.

Of course it would be a serious limitation if this philosophical viewpoint is restricted to the characterization of Minkowski spacetime QFT. Despite all the progress with QFT in CST in relation with the formulation of the local covariance principle, it is too early for such questions involving modular theory. As a preliminary test one could ask whether the diffeomorphisms in Diff(S^1) beyond the Moebius transformations in a chiral theory, which as well-known do not leave the vacuum state invariant, have their origin in modular theory. It is clear that in order to achieve this one has to be more flexible with states, i.e. using also other than the vacuum state and not insisting in a modular automorphism of being globally geometric. In this connection it is very encouraging that recently the idea of local covariance found a satisfactory expression; without having a precise description of this crucial principle, there would not be much chance to make headway with modular localization methods in the CST setting of QFT.

The restriction of globally pure state (vacuum, particle states) to causally localized subalgebras $\mathcal{A}(\mathcal{O})$ leads to thermal KMS states associated with the modular Hamiltonian associated to $(\mathcal{A}(\mathcal{O}),\Omega)$. Modular Hamiltonians give rarely rise to geometric movements (diffeomorphisms). Although outside conformal QFT there is no compact localization region in Minkowski spacetime which leads to a fully *geometric* modular theory, there are rather convincing arguments that the modular automorphism becomes geometric at the horizon of the localization region. The reason is that the holographic projection onto the horizon is a (transverse extended) chiral theory.

In case of timelike Killing symmetries in CST there may even exist an extensions of the spacetime and a state on it such that the modular group of its restriction is identical to the Killing group[28].

One of the most mysterious aspects of localization-caused thermal behavior is its possible connection to ordinary i.e. heat bath thermality. The problem is often referred to as the inverse Unruh effect because one wants to know whether there exists a heat bath thermal system which can be viewed as arising through restricting the vacuum on an extended system i.e. in the spirit of an Unruh effect. This is indeed possible for chiral theories and the connection between the two systems is a conformal transformation which maps the standard volume (here length) factor into the logarithm of the splitting length. Although in higher dimension there is no rigorous argument which relates the two kinds of thermal behavior, there is an educated guess.

Apart from the case of chiral theories on the lightray, where the validity of the *inverse Unruh effect* permits to transform the heat bath volume (here length) law into a logarithmic divergence in $\Delta R \to 0$, a relation between heat bath- and localization-caused entropy is unknown. For higher dimensional localization entropy the split property suggests to consider the entropy of a light sheet whose thickness is the split distance ΔR. By generalizing the localization entropy of chiral theories and by relying on the vacuum polarization divergence of a

[28]The standard example is the Hartle-Hawking state on the Kruskal extension restricted to the region outside a black hole.

dimensionless charge we presented a formula for the leading divergence of the localization entropy and gave strong arguments in favor of a universality between heat bath and localization-caused entropy. From an algebraic viewpoint one parametrizes the approximation of a monad by type I algebras in two different way; in the thermodynamic limit by a L^3 proportional sequence and in the funnel limit one of the length factors was replaces by a log so that effectively one obtains a L^2 (area) proportionality. The monade itself is structureless and the parametrization only appears in the context if its use inside a physical theory.

A ΔR independent area law as that of Bekenstein, in which the light sheet width ΔR is replaced by the Planck length, is not compatible with the localization structure of QFT which requires the quadratic increase. There is a tight connection between modular localization and the phasespace density of states. Whereas the phase space density in QM is finite, that of QFT is *nuclear.* If one interprets the Bekenstein area law as coming from a future quantum theory of gravity (QG) without standard quantum matter, the algebraic structure of such a theory must be that of a very low phasespace density, as e.g. an unknown QM or a combinatorial algebra with trace states. On the other hand the localization entropy from QFT is the precise entropical counterpart within the thermal setting of the Hawking radiation which does not need any appeal to a yet unknown QG. Hence their remains a basic clash between Hawking radiation, which Hawking derived from QFT localized outside a black hole horizon, and the Bekenstein formula which was inferred from interpreting a certain classical area formula.

In the article we also presented examples of unnatural QFT in which the phase space density is too high or too low. Among the unphysical consequences are: the existence of a Hagedorn temperature or the absence of any thermal state, as well as serious problems with causal propagation. In particular they cannot occur in causally propagating situations as formally described by Lagrangian quantization. The algebraic approach to QFT from its very beginning [4][5] tried to isolate them. Their only use has been to exemplify those unphysical properties which a non-Lagrangian approach must avoid and understand those properties which one must require to exclude pathologies. They occur in infinite component QFT as string theories[29]. One also meets them in the AdS_n-CFT_{n-1} correspondence, an explicit illustration is provided by taking a free massive AdS field which on the conformal side yields a generalized conformal field with the mentioned pathologies. Unfortunately the old insights into what constitutes a natural QFT outside the Lagrangian protection have been lost on protagonists of the supersymmetric N=4 Yang Mills – super gravitation AdS model. The attempt to remind them of the problems in their conjectures has remained without avail [8].

The Bekenstein thermodynamical interpretation of a certain quantity in the setting of classical gravity raises the question whether it is not possible to invert this connection i.e. to supplement the thermodynamical setting by reasonable

[29]Contrary to its name, the result of the canonical quantization of the Nambu-Goto Lagrangian is not string-localized but represents a point-localized *dynamical infinite component QFT* [44][1].

assumptions of a general geometric nature, so that the Einstein Hilbert equations are a consequence of the fundamental laws of thermodynamics. Modular theory already relates thermal behavior with localization, hence a relation of fundamental laws of thermodynamics with gravity is not as unexpected as it looks at first sight. The reader is referred to some very interesting observations by Jacobson [49].

Whereas there is hardly any doubt that apart from problems of improved formulations the QM-QFT interface had reached its conceptual final position this is not the case with the interface between QFT in CST and QG. Up to recently the general belief was that the background independence and the entropical area proportionality are marking this interface. But in both cases this had to be amended. On the one hand the new local covariance principle shows that local covariance implies at least the unitary quantum equivalence of QFT in spacetime regions which are isometric[30] which is a big step in the direction of background independence. And if one ignores the logarithmic factor the area proportionality by itself cannot be characteristic for QG.

Returning to the main point in part I; there are hardly two concepts which are that different than relativistic QM and relativistic QFT. In textbooks this is consistently overlooked probably as a result of believing that because they share the Lagrangian quantization formalism and $\hbar$ the only difference is taken care of by adding the word "relativistic". In both parts of this work we explained the difference in terms of the different localization concepts which in turn is intimately related to the difference in the cardinality of phase space degrees of freedom (finite in QM, "nuclear" in QFT [44]). The ignoration or misunderstandings of these differences has been the cause of a major derailment of particle physics [44]: string theory and the Maldacena conjecture. But modular localization also led to the first existence proof for certain interacting field theories (factorizing models) after 80 years of QFT, and it promises to revolutionize gauge theories [52]. In addition it generates the concepts which are necessary two "split" causally separated regions so that the notion of entanglement can also be used in QFT where it has thermal consequences (localization-caused thermal behavior).

Since the crisis of particle physics originated from confusing the *holistic* aspects of modular localization in QFT [44][1][53] in the aftermath of S-matrix theory in the 60's and has solidified ever since, creating a quite misleading intuition even in present day QFT, the only way out is to correct this incorrect understanding of the most important concept which constitutes the essence of QFT. This would have been possible in earlier times when particle physics was done by individuals or represented "schools" of thought. Whether the correction of something which for several decades had the blessing of globalized communities and has already solidified is possible et all, remains to be seen.

[30] It is not clear whether the stronger form of background independence, in which the isomorphism is replaced by an identity, can be achieved.

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